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LETTER TO THE EDITOR

The Lyapunov exponent for magnons in binary spin-glass chains

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Abstract. We present a calculation of the Lyapunov exponent for transverse magnon modes in a Heisenberg spin-glass chain with positive and negative exchange interactions, both analytically as a microcanonical average and also numerically in the usual canonical statistical ensemble. In the positive-ω regime we find that the localization length obeys $\xi(\omega) \propto \omega^{-1}$ at $\omega \to 0$ if $p > q$, where p, $q = 1 - p$ are the $+1$ and -1 spin concentrations, respectively. It is shown that the previously reported $\xi(\omega) \propto \omega^{-2/3}$ singularity, related to the anomalous dispersion law $\omega \propto k^{3/2}$, holds only in the limit of equal concentrations $p = q = 0.5$. The corresponding densities of states diverge as $\rho(\omega) \propto \omega^{-1/2}$ and $\propto \omega^{-1/3}$ for $p > q$ and $p = q$, respectively.

Anderson localization [1] is expected for all eigenstates and any amount of disorder in one-dimensional random chains. One arrives at this result by exploring the statistics of the corresponding product of random transfer matrices whose exponential divergence allows the estimation of the characteristic Lyapunov exponent [2, 3]. Its real and imaginary parts give the inverse localization length and the density of states (DOS), respectively. There is a positive Lyapunov exponent corresponding to a finite localization length for all states in one-dimensional systems with independent site diagonal randomness [2, 3]. However, the localization length shows a strong singularity at zero energy in magnon or phonon studies, which implies delocalization at long wavelengths even in one dimension [4]. In this letter we study the transverse magnon modes at zero temperature [5] in random Heisenberg spin-glass chains with discrete binary distribution of the exchange interaction and focus on the delocalization behaviour at small frequencies.

Interesting results for the magnon localization in spin-glass chains with a symmetric (+ or −) distribution have recently been obtained [5, 6]. The localization length for the long-wavelength eigenstates is weakly diverging and obeys the anomalous $\xi(\omega) \propto \omega^{-2/3}$ law at $\omega \to 0$. The corresponding DOS is $\rho(\omega) \propto \omega^{-1/3}$, which relates to the anomalous $\omega \propto k^{3/2}$ dispersion. The wavenumber k may be regarded as the inverse localization length, leading to the expected singularity for small k. A connection of the magnon problem with the one-dimensional random tight-binding chains near the band edge allowed the exact evaluation of the scaling forms, including the prefactor, relying on conventional perturbational techniques [6]. Earlier [7], a computer simulation study of the closely related isotopically disordered

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harmonic crystal with positive and negative masses also revealed indications for a delocalized mode at zero frequency in one dimension. Mobility edges were reported for that model in two and three dimensions, depending on the concentration.

The purpose of this letter is to determine the characteristic Lyapunov exponent as a function of energy for the random magnet also by choosing an asymmetric exchange distribution. The calculation proceeds in the usual fashion [2] but our model is also suitable for a recently developed microcanonical-ensemble approach [8]. Within the latter method one derives an analytical formula for the Lyapunov exponent in random binary sequences by taking averages in the microcanonical ensemble where the numbers of each matrix type in the product are fixed. Using both of the above methods we have carried out a thorough study of the DOS and the localization length as a function of frequency for different concentrations. Our main result is that for an asymmetric exchange distribution the DOS has a pure $\omega^{-1/2}$-like singularity at low frequencies. We demonstrate that delocalization occurs for the $\omega = 0$ mode via the presence of a $1/\omega$ singularity for the localization length. However, we find that the Lyapunov exponent shows a non-analytical behaviour for $p \to 0.5$ where the corresponding singularities are of different nature.

We shall discuss a tight-binding Hamiltonian corresponding to the Heisenberg spin glass in which the spins occupy a regular one-dimensional lattice [2], expressed by the following simple difference transfer-matrix equation

$$\begin{pmatrix} c_{n+1} \\ c_n \end{pmatrix} = \begin{pmatrix} 2-\xi_n\omega & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} c_n \\ c_{n-1} \end{pmatrix} \qquad n = 0, 1, 2, \ldots, N \tag{1}$$

where ω is the magnon frequency divided by the exchange constant J, and c_n the wavefunction amplitude on the nth site. ξ_n is taken to be an independent random variable with values $+1$ or -1 with probabilities p and $q = 1 - p$, respectively. The object of our study is the asymptotic behaviour of the random-matrix product $\prod_{n=1,N}^{N} \mathbf{M}_n$, where $\mathbf{M}_n$ are the random 2×2 matrices defined by equation (1). There are only two types of transfer matrices in the random product: $\mathbf{M}_+$ with probability p and $\mathbf{M}_-$ with probability $q = 1 - p$. The corresponding Lyapunov exponent is

$$\gamma = \lim_{N\to\infty} \frac{1}{N} \quad \log \frac{\prod_{n=1,N}^{N} \mathbf{M}_n z(0)}{z(0)} \tag{2}$$

using a generic starting vector condition

$$z(0) = \begin{pmatrix} c_1 \\ c_0 \end{pmatrix}.$$

For $p = 1$ we obtain the pure $+1$ chain dispersion $\omega = 2(1 - \cos k)$. Then by adding in the pure $+1$ chain a single -1 impurity from the corresponding scattering problem we obtain the reflection probability

$$R^2 = \frac{4\omega^2}{4\omega^2 + 4\sin^2 k}. \tag{3}$$

Expressed as a function of ω it becomes $R^2 = 4\omega/(3\omega + 4)$, so it can immediately be seen that the $\omega = 0$ mode is reflectionless ($R^2 = 0$). We may use the definition [3] $\xi(\omega) = 2/R^2$, which is valid for small R^2, to obtain the small-ω behaviour

$$\xi(\omega) = \tfrac{2}{3} + 2/\omega. \tag{4}$$

We find that the $1/\omega$ singularity implied by equation (4), which should be valid as $p \to 1$, also holds for any $p > q$. It is violated only at the equal concentration limit $p = q = 0.5$ where it becomes $1/\omega^{2/3}$ instead.

Our results for the Lyapunov exponent obtained using the usual canonical ensemble rely on the application of a theorem [3] which guarantees self-averagability, that is the values of γ from equation (1) for a sufficiently long system converge to the mean values. From the real part of the Lyapunov exponent, taking its inverse, we obtain the localization length. Using an eigenvalue-counting theorem [4], the integrated density of states (IDOS) is directly determined. The corresponding DOS can be obtained in a histogram form by differentiating the IDOS using a finite energy step. The results for the DOS and the localization length obtained by this straightforward numerical method are shown in figure 1. The DOS consists of a positive-ω part containing pN modes and a negative-ω part containing qN modes. It is sufficient to consider only the positive-ω part, which is the same as the negative-ω part for $p < q$. If $p = 1$, all the modes belong to the positive-ω part and we recover the usual one-dimensional DOS. In figure 1(a) we observe that for $p = 0.9$ the DOS already exhibits structure *implying localized states for* ω *not coinciding with* $\omega = 0$, which becomes more pronounced for lower p, e.g figure 1(b). The negative-ω DOS consists of only localized states and is not shown in the figure. Both the DOS and the localization length diverge at $\omega \to 0^+$ if $p > q$ (figure 1(a), (b)). They follow $\rho(\omega) \propto \omega^{-1/2}$ and $\xi(\omega) \propto \omega^{-1}$, respectively. It is only for the mostly disordered $p = q = 0.5$ case (figure 1(c)) where the previously reported [5, 6] anomalous laws hold.

We have verified the above mentioned singularities by performing additional numerical multiplications of up to 10^8 transfer matrices. The results are displayed in figure 2(a), (b). We find that the variation of the concentration does not affect the form of the dominant singular behaviour, already derived for p close to one from the one-impurity calculation, and the new laws appear only at equal concentration.

The mean value of the Lyapunov exponent can be also computed by taking the microcanonical ensemble average, as suggested in [8]. In this case only the position of the M_+ and M_- matrices in the random product is allowed to fluctuate from sample to sample. The approximate Lyapunov exponent $\tilde{\gamma}(p)$ is subsequently given by the formula

$$\tilde{\gamma}(p) = q\ \ln(q) + p\ \ln(p) + \ln\lambda_1(\tilde{x}) - q\ \ln(\tilde{x}). \qquad (5)$$

The saddle point $\tilde{x}$ is derived from the equation

$$\tilde{x}\frac{\partial\lambda_1(\tilde{x})}{\partial\tilde{x}} = q\lambda_1(\tilde{x}) \qquad (6)$$

and the function $\lambda_1(\tilde{x})$ is determined from the largest eigenvalue of the matrix $\mathsf{M}_+ + \mathsf{M}_-\tilde{x}$. Averaging in the microcanonical ensemble involves exactly pN matrices of type $+$ and qN matrices of type $-$ in the different realizations of the product of N matrices, but the sample-to-sample fluctuations due to the random positions of the $+1$ and -1 types of matrices in the product are included. Our results for the localization length obtained via this method are in reasonable qualitative agreement with the corresponding results of figure 1. They give the correct singularity forms for $p > 0.5$ and $p = 0.5$. Using this method we show in figure 3 the expected $1/\omega$ localization-length singularity for the unequal and the $1/\omega^{2/3}$ for the equal

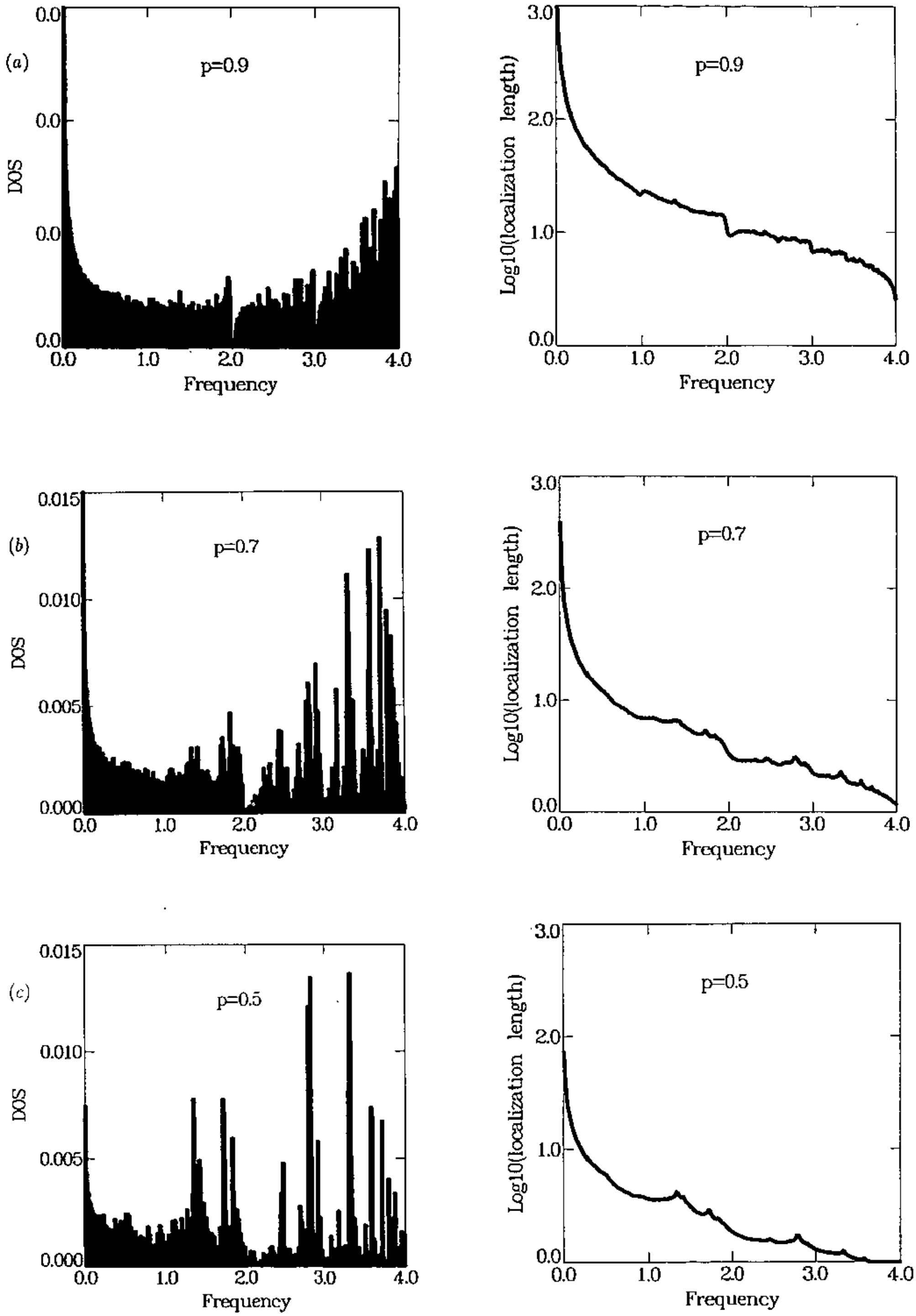

Figure 1. The numerically computed positive-ω averaged density of states (DOS) and the localization length for a random -1 and $+1$ spin-glass chain with $p \geqslant q$. The data are obtained from 4×10^6 long chains and in discrete energy values: (a) $p = 0.9$, (b) $p = 0.7$, (c) $p = 0.5$.

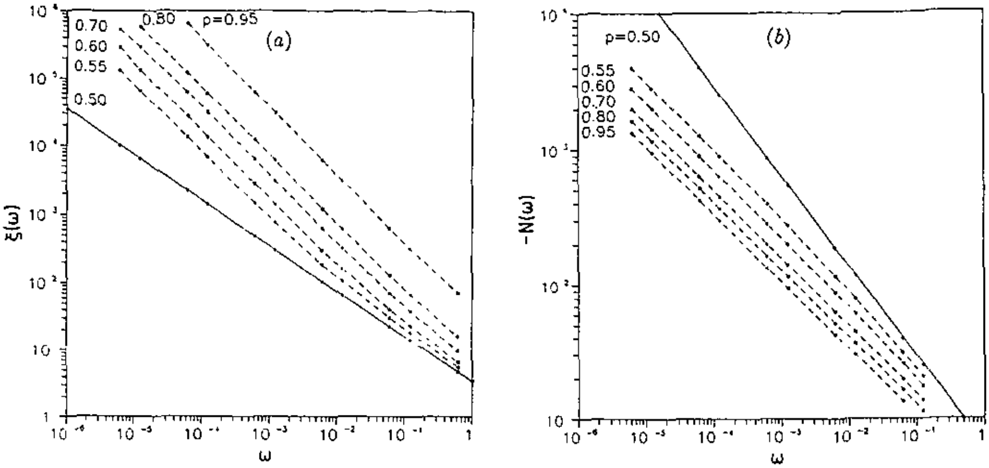

Figure 2. (a) The numerically computed localization length $\xi(\omega)$ in a log–log plot for various concentrations p. The slope is one for $p > 0.5$ and becomes $\frac{2}{3}$ for $p = 0.5$, implying $\xi(\omega) \propto \omega^{-1}$ and $\xi(\omega) \propto \omega^{-2/3}$, respectively. The exact perturbation result $\xi(\omega) = 0.2893\omega^{-2/3}$ is also displayed for $p = 0.5$ with the continuous line. (b) As in (a) but for the inverse of the integrated DOS. The exact perturbation result $N(\omega) = 0.1495\omega^{2/3}$, valid for $p = 0.5$, is also demonstrated. For $p > 0.5$ the behaviour $N(\omega) \propto \omega^{1/2}$ is shown.

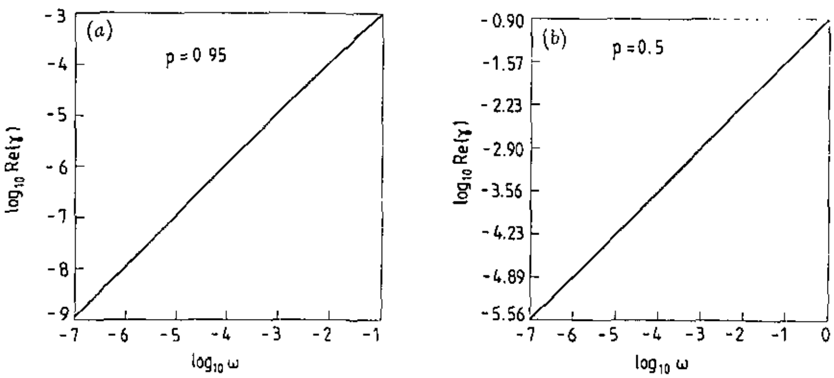

Figure 3. (a) The inverse localization length $\mathrm{Re}(\bar{\gamma})$ versus energy in a log–log plot obtained by the microcanonical ensemble method (equation (5)) for $p = 0.95$. The expected $\xi(\omega) \propto \omega^{-1}$ singularity for $p > q$ appears as a straight line in this figure with slope 1. (b) The same as in (a) but for $p = 0.5$ where the slope is $\frac{2}{3}$ implying a $\xi(\omega) \propto \omega^{-2/3}$ singularity.

concentration cases, respectively. However, with this method we could not obtain the correct prefactors in front of the singularities.

In summary, we have studied the localization length and the DOS for a spin-glass model with binary-type exchange interaction (+ or −), using the usual transfer-matrix techniques as well as a new analytical method. The numerical results obtained are displayed as a function of energy and show convincingly that, for unequal concentrations, the DOS exhibits a singularity near the band centre of exactly the same form as that of the pure chain at the band edge. The localization length also shows a $1/\omega$ singularity peak corresponding to delocalized states at $\omega = 0$ while the rest of the states are localized. The behaviour drastically changes at equal concentrations where the localization-length singularity weakens and takes the previously reported form [5, 6] $\xi(\omega) \propto \omega^{-2/3}$, while the corresponding DOS obeys $\rho(\omega) \propto \omega^{-1/3}$. In the latter case the model exhibits an anomalous dispersion at small frequencies [5]. In conclusion, we focused on the conditions under which partial delocalization occurs for random spin-glass chains via weak $1/\omega$ or $1/\omega^{2/3}$ singularities of the localization length, implying a non-analytical behaviour as $p \to 0.5$. These results should have implications for the corresponding problem of quantum dynamics in random chains [4]. An interesting question is the extension to higher dimensions, where the phenomenon could be further enhanced.

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References

[1] Anderson P W 1958 *Phys. Rev.* **109** 1492
[2] Ishii K 1973 *Suppl. Prog. Theor. Phys.* **53** 77
[3] Lifshits I M, Gredeskul S A and Pastur L A 1988 *Introduction to the Theory of Disordered Systems* (New York: Wiley)
[4] Evangelou S N and Katsanos D E 1992 *Phys. Lett.* A at press
[5] Stinchcombe R B and Pimentel I R 1988 *Phys. Rev.* B **38** 4980
[6] Boukahil A and Huber D L 1989 *Phys. Rev.* B **40** 4638
[7] Canisius J and Van Hemmen J L 1981 *Phys. Rev. Lett.* **46** 1487
[8] Deutsch J M and Paladin G 1989 *Phys. Rev. Lett.* **62** 695